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A NOTE ON THE SPACING POLICIES FOR VEHICLE PLATOONS WITH PREDECESSOR-FOLLOWER AND RING INFORMATION FLOW

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ABSTRACT

In this work, we consider the effects of multiple vehicle look ahead in a vehicle platoon that employs a constant spacing policy (CSP) and a constant time headway policy (CTHP) based controller in the presence of actuator lags and time delays. We demonstrate string instability with a CSP when information from 'r' preceding vehicles is used. In the case of CTHP, information from the "r" preceding vehicles helps increase the tolerance to actuation lags by a factor of "$(r+1)/2$" for the same time headway employed. We further consider a single vehicle CTHP controller with time delays and obtain the relationship between controller gain, time headway and actuator lag or time delay in the system. These relations can aid in the design of CTHP controllers that ensure string stable platoons.

INTRODUCTION

Automated Highway Systems (AHS) have been investigated widely over the past several decades. The motion of each vehicle is controlled such that a desired inter-vehicular spacing is maintained. The spacing policy employed specifies the spacing between each successive pair of vehicles and a controller helps regulate the motion of a vehicle based on motion information from one or more predecessor vehicles. Typically, vehicles in a platoon follow their immediate predecessor using on-board sensory information such as its speed, following distance, relative speed with respect to its immediate predecessor or Vehicle to Vehicle (V2V) or Infrastructure to Vehicle (I2V) communication [1].

Many spacing policies have been proposed in the literature [2–5], which primarily fall into two categories. A Constant Spacing Policy (CSP) where each vehicle input is computed to maintain a constant inter-vehicular spacing irrespective of the velocity of the vehicles in the platoon. Several types of CSP based controllers are discussed in [3], such as a basic CSP controller, CSP controller with acceleration feedback, CSP with multiple vehicle feedback, etc. CSP based controllers may lead to inter vehicular collisions during velocity maneuvers, especially in the absence of lead vehicle information. An alternative to this is a variable spacing policy, such as a Constant Time Headway Policy (CTHP) [2], where the desired following distance varies affinely with its speed and the constant of proportionality is referred to as the time headway. A significant advantage of employing CTHP is that it does not require reference/lead vehicle information for ensuring string stability. Since parasitic actuation/sensing lags are omnipresent in any control system, the problem of how actuation/sensing lags affect string stability was considered first in [3]; it was shown that a parasitic lag of τ imposes a lower bound of 2τ on the employable time headway in a platoon of vehicles. Other spacing policies in the literature include a constant safety factor policy [4], nonlinear spacing laws [6], and a delay based spacing policy discussed in [5]. Since CSP and CTHP are commonly used control laws, we will restrict our attention to these two policies.

In [3], simulations were used to demonstrate that a platoon using CSP based controller with multiple predecessor vehicle feedback is string unstable; no analysis was provided. In this work, we demonstrate string instability using CSP based controllers with every vehicle having information of its "r" predecessors. Our approach considers a realistic model of a vehicle subject to parasitic lags and examining how the parasitic lag leads to string instability. We study the following inverse problem: for a known value of lag and/or delay in the system, we find the range of control and system parameters that result in a string stable platoon.

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In this work, we consider vehicle platoons with two specific information flow graphs, predecessor follower (PF) and ring. We adopt these information flow graphs because (1) the PF information flow has been commonly employed in platooning applications due to its simplicity and (2) the ring information flow is suitable for analytical determination of controller gains and time headway as it is possible to analytically determine the eigenvalues of the closed loop system [7, 8]. The paper is organized as follows. First, we determine conditions for string instability for a 'r' vehicle look ahead CSP controller with acceleration feedback. Then, we investigate conditions for string stability for a 'r' vehicle look ahead CTHP controller with actuator lag to find a relation between the lag and the time headway. Finally, we study a CTHP controller with time delay for a single vehicle look ahead ($r = 1$) and determine the control gains and time headway that will lead to a string stable platoon for a known delay.

PROBLEM BACKGROUND

Spacing Policies

Consider each vehicle as a point mass whose position satisfies the double integrator dynamics

$$\ddot{x}_i = u_i \tag{1}$$

where x_i is the position of the i^{th} vehicle and u_i is the input to the i^{th} vehicle which depends on the type of spacing policy employed, CSP and CTHP. An example of a controller using CSP is given by [3]:

$$u_i = -k_v(\dot{x}_i - \dot{x}_{i-1}) - k_p(x_i - x_{i-1} + L_i) \tag{2}$$

where k_v and k_p are positive controller gains and L_i is the inter-vehicular spacing. The spacing error is defined as $e_i = x_i - x_{i-1} + L_i$. Using the control law in (1), the governing equation for the spacing error is given by $\ddot{e}_i + k_v\dot{e}_i + k_p e_i = k_v\dot{e}_{i-1} + k_p e_{i-1}$ and the transfer function from the $(i-1)$-th spacing error to the i-th spacing error is given by $H(s) = \frac{k_v s + k_p}{s^2 + k_v s + k_p}, \forall\ i \geq 2$. For sufficiently small frequencies ($0 < \omega < \sqrt{2k_p}$), $|H(j\omega)| \geq 1$. Hence, the system is not string stable. A variation of the above controller is obtained by using acceleration feedback term $k_a\ddot{x}_{i-1}$ in the controller, which results in a weakly string stable system when $k_a = 1$ [3]. Using the same controller structure, it was reported that when information from 'r' vehicles ahead is used, only a "weak" sense string stability can be guaranteed. In addition, CSP based controllers lack robustness to actuator lags.

The second spacing policy we consider is the CTHP where the vehicle maintain a constant time headway with respect to their predecessor. Time headway is defined as the time required for the follower vehicle to reach the position of the predecessor vehicle (i.e., the time taken to cover $x_i - x_{i-1} + L_i$). In CTHP the inter vehicular spacing is a function of the velocity of the vehicles in the platoon. Let λ be a positive gain, h be the time headway and $\delta = (x_i - x_{i-1} + L_i + h\dot{x}_i)$ be the generalized spacing error. We consider the following simple CTHP controller [2, 3]:

$$u_i = -\frac{(\dot{x}_i - \dot{x}_{i-1}) + \lambda\delta}{h}. \tag{3}$$

Using this controller the generalized spacing error governing equation is $h\ddot{\delta}_i + (1 + \lambda h)\dot{\delta}_i + \lambda\delta_i = \dot{\delta}_{i-1} + \lambda\delta_{i-1},\quad \forall i \geq 2$ and the associated transfer function is $H(s) = \frac{s+\lambda}{hs^2 + (1+h\lambda)s + \lambda}$, which is string stable $\forall\ h > 0$. Further, the system is shown to possess robustness up to certain values of parasitic lag.

CTHP With Ring Information Flow Graph

When a PF type graph is employed for communication, the resulting closed-loop system matrix cannot be analytically decomposed. However when a ring type communication is employed the resulting closed-loop system matrix is Circulant which can be readily decomposed to study the eigenvalues of the system. This section provides a brief introduction to a platoon with ring communication employing a CTHP based controller.

A simple ring type of information flow graph over a vehicle platoon is shown in Figure 1 [7]. Consider the simple CTHP control input $u_i = K(x_{i-1} - x_i - L_i - h\dot{x}_i)\ \forall\ i \in N$ where $K > 0$ is the gain. This control input yields the following closed-loop dynamics for a platoon consisting of N vehicles:

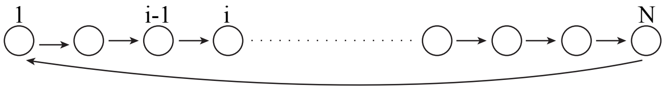

Figure 1. Vehicle platoon with ring communication graph

$$\dot{\mathbf{x}} = A\mathbf{x} + [0, L_1, 0, L_2, \cdots, 0, L_N]^T \tag{4}$$

where $\mathbf{x} = \begin{bmatrix} x_1 \; \dot{x}_1 \; x_2 \; \dot{x}_2 \; \cdots \; x_N \; \dot{x}_N \end{bmatrix}^T$,

$$A = circ\left(\begin{bmatrix} 0 & 1 \\ -K & -hK \end{bmatrix}, O_2, O_2, \cdots, \begin{bmatrix} 0 & 0 \\ K & 0 \end{bmatrix} \right),$$

$circ()$ denotes a circulant matrix, and O_2 is a 2×2 null matrix. To simplify the notation, let $M_1 = \begin{bmatrix} 0 & 1 \\ -K & -\gamma \end{bmatrix}$, $M_2 = \begin{bmatrix} 0 & 0 \\ K & 0 \end{bmatrix}$ and $\gamma = hK$. Since the closed loop system matrix is circulant, it can be block diagonalized with each diagonal element A_i given by $A_i = \begin{bmatrix} 0 & 1 \\ K(\sigma^{(1-i)} - 1) & -\gamma \end{bmatrix}$ where $\sigma = e^{2\pi j/N}$. The eigenvalues of the system are the union of the eigenvalues of the diagonal matrices A_i and are given by

$$\lambda_{i,\pm} = -\frac{\gamma}{2} \pm \frac{\sqrt{\gamma^2 + 4K(\sigma^{(1-i)} - 1)}}{2} \quad i = 1:N. \tag{5}$$

For $i = 1$, the two eigenvalues are zero and $-\gamma$. For $i \in [2, N]$, the real parts of the eigenvalues lie between zero and $-\gamma$ and are symmetric the real axis and the line parallel to the imaginary axis and passing through $-\gamma/2$ if $K < \gamma^2 / \left(2\cos^2 \frac{\pi}{N}\right)$.

MAIN RESULTS

CSP with 'r' Vehicle Look Ahead

When a platoon consists of a large number of vehicles, one can use the information from multiple predecessor vehicles. Let each vehicle have information of 'r' vehicles that are directly ahead of it. The control input corresponding to the CSP can then be written as

$$u_i = \sum_{l=1}^{r} \left(k_{al}\ddot{x}_{i-l} - k_{vl}(\dot{x}_i - \dot{x}_{i-l}) - k_{pl}\left(x_i - x_{i-l} + \sum_{p=\max[0,i-l+1]}^{i} L_p \right) \right) \tag{6}$$

where the subscript $i-k$ denotes the variable between vehicle i and l. Note that for the PF information flow, if $i <= r$, then the control input utilizes only information from $i-1$ vehicles.

In the presence of parasitic lags, the vehicle model may be described as:

$$\begin{aligned} \ddot{x}_i &= f_i, \\ \tau\dot{f}_i + f_i &= u_i, \end{aligned} \tag{7}$$

where f_i is the force per unit mass acting on the i^{th} vehicle and is a filtered version of the control input u_i. The parasitic lag τ is usually unknown but may be bounded, i.e., $\tau \in [0, \tau_0]$ where $\tau = 0$ corresponds to instantaneous actuation and τ_0 is the maximum possible value of parasitic lag. From the point of view of robustness of string stability, any vehicle following law must guarantee string stability for every value of parasitic lag lying between 0 and τ_0. We define the platoon to be robustly string stable if it is string stable for every $\tau \in [0, \tau_0]$ for some $\tau_0 > 0$. The following theorem summarizes the result of this section:

Theorem 1. *A platoon of N vehicles with individual vehicle dynamics given by* (7) *and with the control input given in equation* (6) *is not robustly string stable for any finite r.*

Proof. The governing equation for the spacing error dynamics is given by

$$\tau\frac{d}{dt}\ddot{e}_i + \ddot{e}_i + \sum_{l=1}^{r}(k_{vl}\dot{e}_i + k_{pl}e_i) = \sum_{l=1}^{r}\left(k_{al}\ddot{e}_{i-l} + k_{vl}\dot{e}_{i-l} + k_{pl}e_{i-l}\right). \tag{8}$$

Let $H_l(s) = \frac{k_{al}s^2 + k_{vl}s + k_{pl}}{\tau s^3 + s^2 + \sum_{l=1}^{r}(k_{vl}s + k_{pl})}$. Then, the error equation may be written as

$$E_i(s) = \sum_{l=1}^{r} H_l(s)E_{i-l}(s). \tag{9}$$

If we consider the spacing errors to be states of a spatially discrete system, we have the following characteristic polynomial for the spacing error dynamics when the lead vehicle performs a sinusoidal acceleration maneuver at a frequency ω:

$$P(z) = z^r - \sum_{l=1}^{r} H_l(j\omega)z^{r-k}. \tag{10}$$

There are three cases to consider: (1) $\sum_{l=1}^{r} k_{al} < 1$, (2) $\sum_{l=1}^{r} k_{al} > 1$, and (3) $\sum_{l=1}^{r} k_{al} = 1$. In each of the three cases, we will use an approach that relies on perturbation analysis to show that the platoon is string unstable. In particular, we will show that the root at $z = 1$ moves outside the unit circle for either sufficiently small values of ω or $\frac{1}{\omega}$. Note that since $\sum_{l=1}^{r} H_l(0) = 1$, $P(1) = 0$; thus, there is a root at $z = 1$.

Case (1): We will evaluate how the root at $z = 1$ changes with ω around $\omega = 0$. Since there is no drop in the degree of the polynomial as ω varies, the roots are continuous functions of ω.

Let $z(\omega) = R(\omega)e^{j\theta(\omega)}$. Denote y' and y'' as the first and second derivatives of the variable y with respect to ω. The first and the second derivatives of z with respect to ω at $(\omega = 0, R(\omega) = 1)$ are given by

$$z'(0) = R'(0) + j\theta'(0), \tag{11}$$

$$z''(0) = R''(0) - (\theta'(0))^2 + j(\theta''(0) + 2R'(0)\theta'(0)). \tag{12}$$

Later we will use the Taylor series expansion of $R(\omega)$ around $\omega = 0$, i.e., $R(\omega) = R(0) + \omega R'(0) + (\omega^2/2)R''(0) + h.o.t.$

The derivative of $P(z(\omega))$ with respect to ω is given by setting $P'(z(\omega)) = 0$:

$$P'(z) = rz^{r-1}z' - \sum_{l=1}^{r}\left(H_l' z^{r-l} + H_l(r-l)z^{r-l-1}z'\right).$$

Evaluating $P'(z)$ at ($z = 1$ and $\omega = 0$) and setting it equal to zero we obtain $z'(0) = 0$. This means that $R'(0) = 0$ and $\theta'(0) = 0$. Since this does not provide any indication of how the root changes with respect to ω around $\omega = 0$, we will consider the second derivative of $P(z)$:

$$\begin{aligned} P''(z) = rz^{r-1}z'' + r(r-1)z^{r-2}(z')^2 - \sum_{l=1}^{r}\Big(H_l'' z^{r-l} \\ + 2H_l'(r-l)z^{r-l-1}z' + H_l(r-l)(r-l-1)z^{r-l-2}(z')^2 \\ + H_l(r-l)z^{r-l-1}z''\Big). \qquad (13) \end{aligned}$$

Evaluating $P''(z)$ at ($z = 1$ and $\omega = 0$) and setting it equal to zero we obtain

$$\begin{aligned} rz''(0) - \sum_{l=1}^{r} H_l''(0) - \sum_{l=1}^{r}(r-l)H_l(0)z''(0) = 0 \\ \implies z''(0) = \frac{\sum_{l=1}^{r} H_l''(0)}{r - \sum_{l=1}^{r}(r-l)H_l(0)} = \frac{\sum_{l=1}^{r} H_l''(0)}{\sum_{l=1}^{r-1} lH_l(0)}. \qquad (14) \end{aligned}$$

Let $K_p := \sum_{l=1}^{r} k_{pl}$ and $K_v := \sum_{l=1}^{r} k_{vl}$. From $H_l(j\omega)$ we can determine $H_l(0) = k_{pl}/K_p$, $H_l'(0) = j(k_{vl}K_p - k_{pl}K_v)/K_p^2$, and

$$H_l''(0) = \frac{2(H_l(0) - k_{al}) - 2H_l'(0)K_v j}{K_p}.$$

From this we get $\sum_{l=1}^{r} H_l(0) = 1$, $\sum_{l=1}^{r} H_l'(0) = 0$, and $\sum_{l=1}^{r} H_l''(0) = \frac{2\left(1 - \sum_{l=1}^{r} k_{al}\right)}{K_p}$. Hence, $z''(0)$ is real and positive when $\sum_{l=1}^{r} k_{al} < 1$. Therefore, $R''(0) > 0$ which implies that there is a root with magnitude greater than one for small ω.

Case (2): If $\sum_{l=1}^{r} k_{al} > 1$, then let $\lambda = \frac{1}{\omega}$; further, define $\bar{H}_l(\lambda) := H(\frac{1}{\lambda})$. Clearly, then for $\tau = 0$, we have:

$$\bar{H}_l(\lambda) = \frac{k_{al} + k_{vl}\lambda + k_{pl}\lambda^2}{1 + \sum_{l=1}^{r} k_{vl}\lambda + \sum_{l=1}^{r} k_{pl}\lambda^2}.$$

Writing $P(z)$ in terms of λ, we get $P(z) = z^r - \sum_{l=1}^{r} \bar{H}_l(\lambda)z^{r-l}$.

Let $K_a = \sum_{l=1}^{r} k_{al} > 1$. Consider $\lambda = 0$ and note that $\sum_{l=1}^{r} \bar{H}_l(0) = \sum_{l=1}^{r} k_{al} > 1$. Clearly, $P(1) = 1 - \sum_{l=1}^{r} \bar{H}_l(0) < 0$, and

$$\begin{aligned} P(K_a) = K_a^r - \sum_{l=1}^{r} k_{al}K_a^{r-l} > K_a^r - \sum_{l=1}^{r} k_{al}K_a^{r-1} \\ = K_a^r - K_a^r = 0. \end{aligned}$$

Hence, when $\lambda = 0$, $P(z)$ is a real polynomial and has a real root between 1 and $K_a > 1$ and is not Schur consequently. The above argument presumes $k_{al} > 0$; if this were not the case, then for sufficiently large real values of z, the term z^r dominates other terms and the polynomial $P(z)$ will be positive for all such values. Even in this case, the polynomial $P(z)$ will have a real positive root outside the unit disk. Therefore, when $\lambda = 0$, one root of $P(z)$ lies outside the unit disk. From the continuity of roots with respect to the coefficients, for sufficiently small values of λ (or equivalently, sufficiently large values of ω), the polynomial $P(z)$ has a root outside the unit disk.

Case (3): When $\sum_{l=1}^{r} k_{al} = 1$ and $\tau = 0$, we notice that for every s, $z = 1$ is always a root of

$$P(z) = z^r - \sum_{l=1}^{r} \frac{k_{al}s^2 + k_{vl}s + k_{pl}}{s^2 + \sum_{l=1}^{r} k_{vl}s + \sum_{l=1}^{r} k_{pl}} z^{r-l}.$$

Let us fix *any* s and vary τ and examine the perturbation to this root at $z = 1$ using the approach for case 1. Let y' denote $\frac{dy}{d\tau}$ in this case. In order to make the dependence of $H_l(s)$ on τ explicit, let us write

$$H_l(s, \tau) = \frac{k_{al}s^2 + k_{vl}s + k_{pl}}{\tau s^3 + s^2 + \sum_{l=1}^{r} k_{vl}s + \sum_{l=1}^{r} k_{pl}}.$$

Differentiating the characteristic polynomial with respect to τ yields:

$$rz^{r-1}z' - \sum_{l=1}^{r} H_l(s,\tau)(r-l)z^{r-l-1}z' - \sum_{l=1}^{r} H_l'(s,\tau)z^{r-l} = 0.$$

When $z = 1$, $\tau = 0$, simplifying the above equation yields:

$$z'(0) = \frac{-s^3}{\sum_{l=1}^{r} l(k_{al}s^2 + k_{vl}s + k_{pl})}.$$

When $s = j\omega$, the real part of $z'(0)$ denoted by $\Re(z'(0))$ is given by:

$$\Re(z'(0)) = \frac{\omega^4(\sum_{l=1}^{r} lk_{vl})}{(\sum_{l=1}^{r} l(k_{pl}-k_{al}\omega^2)^2+\omega^2(\sum_{l=1}^{r} lk_{vl})^2} > 0.$$

If we express $z(\tau)$ in the polar form as $R(\tau)e^{j\theta(\tau)}$, then $\Re(z'(0)) = R'(0) > 0$, indicating that for any sufficiently small $\tau_0 > 0$, the root of the characteristic equation will be outside the unit circle. ■

CTHP with ‘r’ Vehicle Look Ahead and Actuator Lag

In this section we will find a bound on the actuator lag in terms of the time headway and ‘r’ which will result in string instability. Consider the following CTHP control law:

$$u_i = -\frac{\sum_{l=1}^{r}(\dot{x}_i - \dot{x}_{i-l} + \lambda\delta_{i-l})}{h}. \tag{15}$$

Then, the bound on the actuator lag is given by the following theorem.

Theorem 2. *When the CTHP control law from* (15) *is used with the vehicle dynamics from* (7)*, then the platoon is string stable for* $\tau_0 \leq h(r+1)/4$.

Proof. The generalized spacing error dynamics are given by

$$h\tau\dddot{\delta}_i + h\ddot{\delta}_i + \sum_{l=1}^{r}\Big((1+rh\lambda)\dot{\delta}_i + \lambda\delta_i\Big) = \sum_{l=1}^{r}\Big(\dot{\delta}_{i-l} + \lambda\delta_{i-l}\Big).$$

The error equation may be written as $E_i(s) = \sum_{l=1}^{r} H_l(s)E_{i-l}(s)$, where

$$\sum_{l=1}^{r} H_l(s) = \frac{r(s+\lambda)}{\tau h s^3 + hs^2 + r[(2+\frac{r+1}{2}\lambda h)s+\lambda]}$$

A sufficient condition for string stability is $\sum_{l=1}^{r}|H_l(j\omega)| \leq 1$. A necessary condition for $\sum_{l=1}^{r}|H_l(j\omega)| \leq 1$ is $|\sum_{l=1}^{r} H_l(j\omega)| \leq 1$. The condition $|\sum_{l=1}^{r} H_l(j\omega)| \leq 1$ can be simplified to the following inequality:

$$\begin{aligned}\tau^2h^2\omega^4 + \omega^2(h^2 - 2r\tau h(1+(r+1)\lambda h/2)&\\ + r^2(1+(r+1)\lambda h/2)^2 - r^2 - 2r\lambda h &\geq 0.\end{aligned}$$

Since $r^2(1+(r+1)\lambda h/2)^2 - r^2 - 2r\lambda h \geq 0$, the worst case condition for satisfying the above relation is $h^2 - 2r\tau h(1+(r+1)\lambda h/2) \geq 0$. We will derive the given actuator lag condition in the theorem from these two conditions:

$$r^2(1+(r+1)\lambda h/2)^2 - r^2 - 2r\lambda h \geq 0 \tag{16}$$
$$h - 2r\tau(1+(r+1)\lambda h/2) \geq 0 \tag{17}$$

We can use (17) in (16) in the following manner to obtain:

$$\begin{aligned}r(1+(r+1)\lambda h/2)^2 &\geq r + 2\lambda h \geq r + 4\lambda r\tau(1+(r+1)\lambda h/2)\\ (1+(r+1)\lambda h/2 - 2\tau\lambda)^2 &\geq 1 + 4\tau^2\lambda^2\\ (1+(r+1)\lambda h/2 - 2\tau\lambda)) &\geq \sqrt{1+4\tau^2\lambda^2} \geq 1\\ \implies h &\geq 4\tau/(r+1) \qquad (18)\end{aligned}$$

This is satisfied when $\tau_0 \leq h(r+1)/4$. ■

Theorem 2 indicates that in the presence of an actuator lag, a lower bound on the time headway h to ensure string stability is given by $4\tau_0/(r+1)$.

CTHP With Time Delay

In this section we study the effect of time delay with a CTHP controller which also includes the relative velocity error. The CTHP based controllers rely on vehicle measurements and communication for position and velocity data, which may be delayed causing loss of stability or performance degradation [8]. The knowledge of maximum time delay that can be tolerated for a given controller and operational parameters could aid in the design of the spacing control laws.

In the presence of a constant delay τ_d, the CTHP controller from (3) may be rewritten as

$$u_i(t) = -\frac{\dot{x}_i(t-\tau_d) - \dot{x}_{i-1}(t-\tau_d) + \lambda\delta(t-\tau_d)}{h}, \tag{19}$$

and the closed-loop dynamics as

$$\dot{\mathbf{x}} = A\mathbf{x}(t-\tau_d). \tag{20}$$

A bound on permissible time delay for a simple CTHP controller without velocity error feedback as in Eq. (19) was derived in [9]. For the system given in (20), if one can determine the eigenvalues, then the worst case delay corresponding to each eigenvalue can be computed and the minimum of these is the maximum allowable delay in the platoon. Since for the ring graph, we can analytically determine the eigenvalues as a function of controller and design parameters, we can obtain a relation between time delay and the parameters. If s is an eigenvalue of the system, then the maximum time delay (τ_d^s) corresponding to that eigenvalue is given by [10]

$$\tau_d^s = \frac{\tan^{-1}(-\Re(s)/\Im(s))}{|s|}. \tag{21}$$

where $\Re(s)$ and $\Im(s)$ are the real and imaginary parts of the eigenvalue s, respectively, and $|s|$ is the magnitude of the eigenvalue.

Theorem 3. *With the ring information flow, the delayed system in* (20) *is stable in the presence of the time delay* τ_d *if*

$$\tau_d < \min\left(\frac{\pi}{2\lambda}, \frac{h}{2}\frac{\theta_{i,N}}{\sin\theta_{i,N}}\right).$$

where $\theta_{i,N} = \frac{\pi(1-i)}{N}$.

Proof. For the ring information flow, the system matrix in (20) is $A = circ(M_1, O_2, \cdots, M_2)$ with

$$M_1 = \begin{bmatrix} 0 & 1 \\ -\lambda/h & (-\lambda - 1/h) \end{bmatrix}, \; M_2 = \begin{bmatrix} 0 & 0 \\ \lambda/h & 1/h \end{bmatrix}.$$

Decomposing the system matrix into its block diagonal form as before, the characteristic equation for the i-th block diagonal matrix is $s^2 + s\left(\lambda + \frac{(1-\sigma^{1-i})}{h}\right) + \frac{\lambda(1-\sigma^{1-i})}{h} = 0$, and the eigenvalues are located at

$$s_{i,(1,2)} = -\lambda, -\frac{1-\sigma^{1-i}}{h}, \quad i = 1, 2, ..N \tag{22}$$

Hence, the delay bound for the eigenvalues at $s_{i,1} = -\lambda$ is $\tau_d^{s_1} = \pi/(2\lambda)$. For the second eigenvalue $s_{i,2}$, the delay bound is given by

$$\tau_d^{s_2} = \frac{h}{2}\left(\frac{\tan^{-1}\left(\frac{1-\cos\frac{2\pi(1-i)}{N}}{\sin\frac{2\pi(1-i)}{N}}\right)}{\sin\frac{\pi(1-i)}{N}}\right) = \frac{h}{2}\frac{\theta_{i,N}}{\sin\theta_{i,N}}.$$

Therefore, the bound on the maximum permissible delay is the minimum of the two delay bounds $\tau_d^{s_1}$ and $\tau_d^{s_2}$, i.e., $\tau_d < \min(\tau_d^{s_1}, \tau_d^{s_2})$ ■

SIMULATIONS

In this section, we discuss numerical simulations that were conducted to verify the results of Theorems 1 and 3.

First, a 15 vehicle platoon is considered to verify the behavior of a CSP controller with multiple vehicle look ahead (6). The following parameter values are used: inter-vehicular spacing $L_i = 5$ m; desired platoon velocity $v = 29$ m/s; PF information flow with $r = 2$. For simplicity the gain values for all the controllers are chosen equal. The evolution of the spacing errors with $\sum_{j=1}^{r} k_{ak}$ equal to 0, equal to 1 ($k_{ak} = 0.5$), less than 1 ($k_{ak} = 0.3$), and greater than 1 ($k_{ak} = 0.6$), respectively, are shown in the four subfigures of Figure 2 from top to bottom. A non-zero parasitic lag of 0.1 s is chosen in the simulations. Zero initial spacing errors are considered with an introduction of a sinusoidal disturbance on the first vehicle at the three second mark. As expected the spacing errors grow unbounded for all the cases considered.

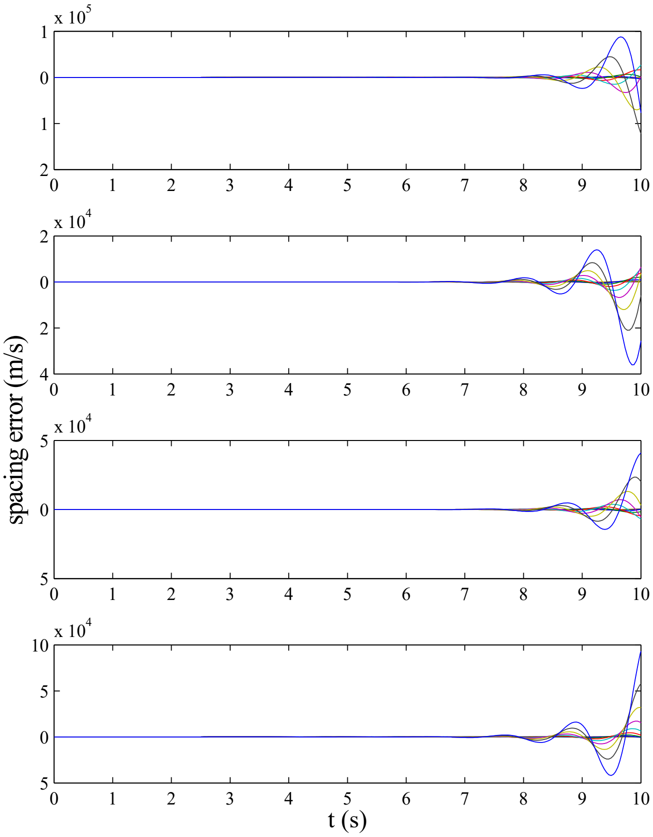

Figure 2. Evolution of spacing errors for $K_a = 0$, $K_a = 1$, $K_a < 1$, and $K_a > 1$ (top to bottom).

Second, numerical simulations were conducted for a 15 vehicle platoon with a CTHP controller in the presence of time delays. From (22), of the $2N$ eigenvalues of an N-vehicle platoon, there are N eigenvalues at $-\lambda$ and the remaining N lie on a closed curve whose equation is given by $ha^2 + hb^2 - 2a = 0$; this is obtained by substituting $s = a + jb$. Figure 3 provides the locations of the closed loop eigenvalues for a 15 vehicle platoon in the absence of any delay; it is clear that all the eigenvalues have negative real parts except for the one zero eigenvalue corresponding to the rigid motion of the platoon. From Theorem 3, the value τ_d^{s2} depends on N. Figure 4 provides the plots of τ_d^{s1} and τ_d^{s2} with N for $h = 0.1$ and $\lambda = 25$. Note that τ_d^{s2} approaches a steady state value of $h/2$. Thus, for large platoons ($N \to \infty$) we can approximate the maximum allowable delay as

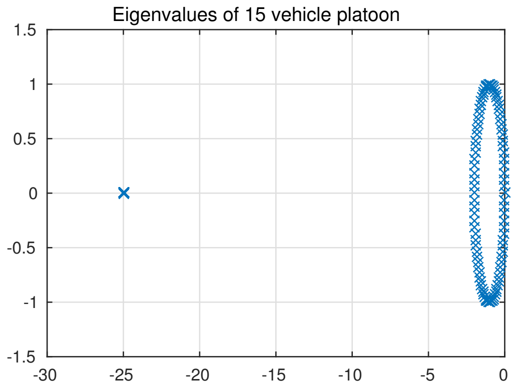

Figure 3. Eigenvalues of a 15 vehicle platoon with ring graph; CTHP controller from (19) with $h = 1$ and $\lambda = 25$

$$\tau_d^* = \min(\tau_d^{s1}, \tau_d^{s2}) = \min\left(\frac{\pi}{2\lambda}, \frac{h}{2}\right). \tag{23}$$

For time headway values satisfying $h < \pi/\lambda$, τ_d^* increases with h until $\tau_d^* = \pi/2\lambda$; beyond this value, the maximum allowable delay remains constant irrespective of h. Figure 5 shows this behavior for ring, PF and leader-PF (LPF) information flows. The parameters used in the simulation are: desired platoon velocity $v = 32$ m/s, gain $\lambda = 25$, and $N = 100$. The time headway h is varied from 0 to 0.15. To draw this plot, we have computed individual eigenvalues of the system matrices for ring, PF and LPF and the allowable delay bound for each eigenvalue is found using (21). Note that for large N, if $\lambda h > \pi$, then the lower bound on the delay is $\frac{\pi}{2\lambda}$, and if $\lambda h < \pi$, then the lower bound on the delay is $\frac{h}{2}$. The relationship in equation (23) may be used in the selection of appropriate gains and a time headway for the platoon.

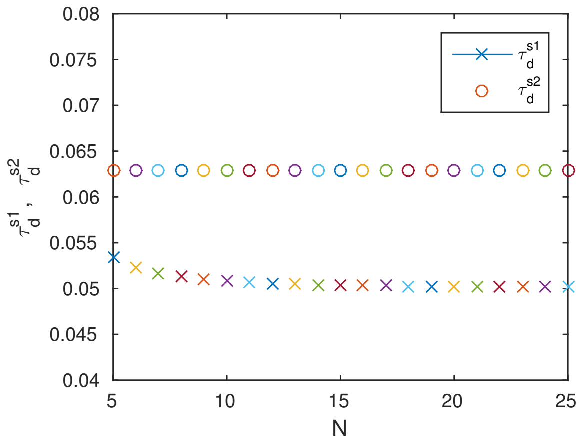

Figure 4. τ_d^{s1}, τ_d^{s2} vs N

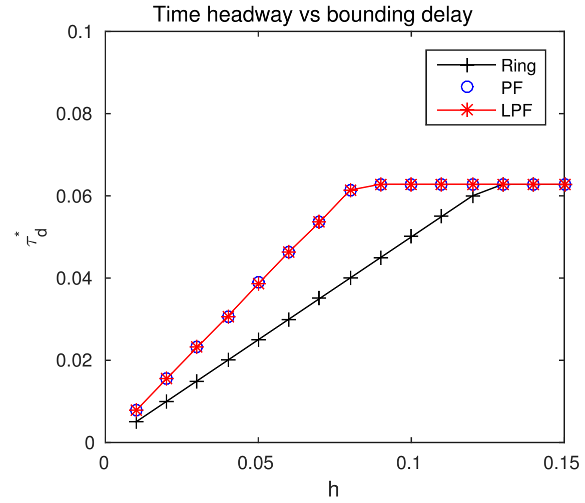

Figure 5. Bounding delay vs time headway for ring, PF and LPF graphs

CONCLUSIONS

In this paper, we have studied the properties of vehicle platoons with CSP and CTHP controllers with information from 'r' preceding vehicles. We have shown that with a 'r' vehicle look ahead CSP controller, the platoon is not robustly string stable consider actuator lags irrespective of the of acceleration feedback gain values. We have also shown that using a CTHP controller with 'r' vehicle look ahead and a given time headway the maximum allowable lag increases with increase in 'r'. In addition, we have determined a bound on the maximum allowable time delay which makes the platoon string unstable; this delay bound depends on the value of the time headway. These bounds may be utilized in the selection of time headway and controller gains.

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